CHAPTER 13

Factors of Safety and Reliability

Factors of safety provide a quantitative indication of slope stability. A value of $F = 1.0$ indicates that a slope is on the boundary between stability and instability; the factors tending to make the slope stable are in precise balance with those tending to make the slope unstable. A calculated value of F less than 1.0 indicates that a slope would be unstable under the conditions contemplated, and a value of F greater than 1.0 indicates that a slope would be stable.

If we could compute factors of safety with absolute precision, a value of $F = 1.1$ or even 1.01 would be acceptable. However, because the quantities involved in computing factors of safety are always uncertain to some degree, computed values of F are never absolutely precise. We need larger factors of safety to be sure (or sure enough) that a slope will be stable. How large the factor of safety should be is determined by experience, by the degree of uncertainty that we think is involved in calculating F, and by the consequences that would ensue if the slope failed.

The reliability of a slope (R) is an alternative measure of stability that considers explicitly the uncertainties involved in stability analyses. The reliability of a slope is the computed probability that a slope will not fail and is 1.0 minus the probability of failure:

$$R = 1 - P_f \tag{13.1}$$

where P_f is the probability of failure and R is the reliability or probability of no failure. A method for computing P_f is described later in the chapter. Factors of safety are more widely used than R or P_f to characterize slope stability. Although R and P_f are equally logical measures of stability, there is less experience with their use, and therefore less guidance regarding acceptable values.

Another consideration regarding use of reliability and probability of failure is that it is sometimes easier to explain the concepts of reliability or probability of failure to laypeople. However, some find it disturbing that a slope has a probability of failure that is not zero, and may not be comfortable hearing that there is some chance that a slope might fail. Factors of safety and reliability complement each other, and each has its own advantages and disadvantages. Knowing the values of both is more useful than knowing either one by itself.

DEFINITIONS OF FACTOR OF SAFETY

The most widely used and most generally useful definition of factor of safety for slope stability is

$$F = \frac{\text{shear strength of the soil}}{\text{shear stress required for equilibrium}} \tag{13.2}$$

Uncertainty about shear strength is often the largest uncertainty involved in slope stability analyses, and it is therefore logical that the factor of safety—called by George Sowers the *factor of ignorance*—should be related directly to shear strength. One way of judging whether a value of F provides a sufficient margin of safety is by considering the question: What is the lowest conceivable value of shear strength? A value of $F = 1.5$ for a slope indicates that the slope should be stable even if the shear strength was 33% lower than anticipated (if all the other factors were the same as anticipated). When shear strength is represented in terms of c and ϕ, or c' and ϕ', the same value of F is applied to both of these components of shear strength.

It can be said that this definition of factor of safety is based on the assumption that F is the same for every point along the slip surface. This calls into question

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whether such analyses are reasonable, because it can be shown, for example by finite element analyses, that the factor of safety for every slice is *not* the same, and it therefore appears that an underlying assumption of limit equilibrium analysis is not true. However, despite the fact that the local factor of safety may be more or less than the value of F calculated by conventional limit equilibrium methods, the average value calculated by these methods is a valid and very useful measure of stability. The factor of safety determined from conventional limit equilibrium analyses is the answer to the question: By what factor could the shear strength of the soil be reduced before the slope would fail? This is a significant question, and the value of F calculated as described above is the most generally useful measure of stability that has been devised.

Alternative Definitions of F

Other definitions of F have sometimes been used for slope stability. For analyses using circular slip surfaces, the factor of safety is sometimes defined as the ratio of resisting moment divided by overturning moment. Because the resisting moment is proportional to shear strength, and the shear stress required for equilibrium of a mass bounded by a circular slip surface is proportional to the overturning moment, the factor of safety defined as the ratio of resisting to overturning moment is the same as the factor of safety defined by Eq. (13.1).

In times past, different factors of safety were sometimes applied to cohesion and friction. However, this is seldom done any more. The strength parameters c and ϕ, or c' and ϕ', are empirical coefficients in equations that relate shear strength to normal stress or to effective normal stress. There is no clear reason to factor them differently, and the greater complexity that results if this is done seems not to be justified by additional insight or improved basis for judging the adequacy of stability.

Reinforcing and anchoring elements within a slope impose stabilizing forces that, like the soil strength, should be factored to include a margin of safety to reflect the fact that there is uncertainty in their magnitudes. The issues causing uncertainties in reinforcing and anchoring forces are not the same as the issues leading to uncertainties in soil strength, and it is therefore logical to apply different factors of safety to reinforcement and soil strength. This can be achieved by prefactoring reinforcement and anchor forces and including them in stability analyses as known forces that are not factored further in the course of the analysis.

FACTOR OF SAFETY CRITERIA

Importance of Uncertainties and Consequences of Failure

The value of the factor of safety used in any given case should be commensurate with the uncertainties involved in its calculation and the consequences that would ensue from failure. The greater the degree of uncertainty about the shear strength and other conditions, and the greater the consequences of failure, the larger should be the required factor of safety. Table 13.1 shows values of F based on this concept.

Corps of Engineers' Criteria for Factors of Safety

The values of factor of safety listed in Table 13.2 are from the U.S. Army Corps of Engineers' slope stability manual. They are intended for application to slopes of embankment dams, other embankments, excavations, and natural slopes where conditions are well understood and where the properties of the soils have been studied thoroughly. They represent conventional, prudent practice for these types of slopes and conditions, where the consequences of failure may be significant, as they nearly always are for dams.

Recommended values of factor of safety, like those in Table 13.2, are based on experience, which is logical. It is not logical, however, to apply the same values of factor of safety to conditions that involve widely varying degrees of uncertainty. It is therefore significant that the factors of safety in Table 13.2 are intended for Corps of Engineers' projects, where methods of exploration, testing, and analysis are consistent from one project to another and the degree of uncertainty regarding these factors does not vary widely. For other situations, where practices and circumstances differ, the values of F in Table 13.2 may not be appropriate.

RELIABILITY AND PROBABILITY OF FAILURE

Reliability calculations provide a means of evaluating the combined effects of uncertainties and a means of distinguishing between conditions where uncertainties are particularly high or low. Despite the fact that it has potential value, reliability theory has not been used much in routine geotechnical practice because it involves terms and concepts that are not familiar to many geotechnical engineers, and because it is commonly perceived that using reliability theory would require more data, time, and effort than are available in most circumstances.

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Table 13.1 Recommended Minimum Values of Factor of Safety

Cost and consequences of slope failure	Uncertainty of analysis conditions: Small[a]	Uncertainty of analysis conditions: Large[b]
Cost of repair comparable to incremental cost to construct more conservatively designed slope	1.25	1.5
Cost of repair much greater than incremental cost to construct more conservatively designed slope	1.5	2.0 or greater

[a]The uncertainty regarding analysis conditions is smallest when the geologic setting is well understood, the soil conditions are uniform, and thorough investigations provide a consistent, complete, and logical picture of conditions at the site.

[b]The uncertainty regarding analysis conditions is largest when the geologic setting is complex and poorly understood, soil conditions vary sharply from one location to another, and investigations do not provide a consistent and reliable picture of conditions at the site.

Table 13.2 Factor of Safety Criteria from U.S. Army Corps of Engineers' Slope Stability Manual

Types of slopes	Required factors of safety[a]: For end of construction[b]	For long-term steady seepage	For rapid drawdown[c]
Slopes of dams, levees, and dikes, and other embankment and excavation slopes[c]	1.3	1.5	1.0–1.2

[a]For slopes where either sliding or large deformations have occurred, and back analyses have been performed to establish design shear strengths, lower factors of safety may be used. In such cases probabilistic analyses may be useful in supporting the use of lower factors of safety for design. Lower factors of safety may also be justified when the consequences of failure are small.

[b]Temporary excavated slopes are sometimes designed only for short-term stability, with knowledge that long-term stability would be inadequate. Special care, and possibly higher factors of safety, should be used in such cases.

[c]$F = 1.0$ applies to drawdown from maximum surcharge pool, for conditions where these water levels are unlikely to persist for long enough to establish steady seepage. $F = 1.2$ applies to maximum storage pool level, likely to persist for long periods prior to drawdown. For slopes in pumped storage projects, where rapid drawdown is a normal operating condition, higher factors of safety (e.g., 1.3 to 1.4) should be used.

Harr (1987) defines the engineering definition of *reliability* as follows: ''Reliability is the probability of an object (item or system) performing its required function adequately for a specified period of time under stated conditions.'' As it applies in the present context, the *reliability of a slope* can be defined as follows: The reliability of a slope is the probability that the slope will remain stable under specified design conditions. The design conditions include, for example, the end-of-construction condition, the long-term steady seepage condition, rapid drawdown, and earthquake of a specified magnitude.

The design life of a slope and the time over which it is expected to remain stable are usually not stated explicitly but are generally thought of as a long time, probably beyond the lifetime of anyone alive today. The element of time may be considered more explicitly when design conditions involve earthquakes with a specified return period, or other loads whose occurrence can be stated in probabilistic terms.

Christian et al. (1994), Tang et al. (1999), Duncan (2000), and others have described examples of the use of reliability for slope stability. Reliability analysis can be applied in simple ways, without more data, time, or

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effort than are commonly available. Working with the same quantity and types of data, and the same types of engineering judgments that are used in conventional analyses, it is possible to make approximate but useful evaluations of probability of failure and reliability.

The results of simple reliability analyses are neither more accurate nor less accurate than factors of safety calculated using the same types of data, judgments, and approximations. Although neither deterministic nor reliability analyses are precise, they both have value, and each enhances the value of the other. The simple types of reliability analyses described in this chapter require only modest extra effort compared to that required to calculate factors of safety, but they can add considerable value to the results of slope stability analyses.

STANDARD DEVIATIONS AND COEFFICIENTS OF VARIATION

If several tests are performed to measure a soil property, it will usually be found that there is scatter in the values measured. For example, consider the undrained strengths of San Francisco Bay mud measured at a site on Hamilton Air Force Base in Marin County, California, that are shown in Table 13.3. There is no discernible systematic variation in the measured values of shear strength between 10 and 20 ft depth at the site. The differences among the values in Table 13.3 are due to natural variations in the strength of the Bay mud in situ, and varying amounts of disturbance of the test specimens. Standard deviation is a quantitative measure of the scatter of a variable. The greater the scatter, the larger the standard deviation.

Statistical Estimates

If a sufficient number of measurements have been made, the standard deviation can be computed using the formula

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{1}^{N}(x - x_{\text{av}})^2} \qquad (13.3)$$

where σ is the standard deviation, N the number of measurements, x the measured variable, and x_{av} the average value of x. Standard deviation has the same units as the measured variable.

The average of the 20 measured values of s_u in Table 13.3 is 0.22 tsf (tons/ft^2). The standard deviation, computed using Eq. (13.3), is

Table 13.3 Undrained Shear Strength Values for San Francisco Bay Mud at Hamilton Air Force Base in Marin County, California[a]

Depth (ft)	Test	s_u (tons/ft^2)
10.5	UU	0.25
	UC	0.22
11.5	UU	0.23
	UC	0.25
14.0	UU	0.20
	UC	0.22
14.5	UU	0.15
	UC	0.18
16.0	UU	0.19
	UC	0.20
	UU	0.23
	UC	0.25
16.5	UU	0.15
	UC	0.18
17.0	UU	0.23
	UC	0.26
17.5	UU	0.24
	UC	0.25
19.5	UU	0.24
	UC	0.21

[a]Values measured in unconfined compression (UC) and unconsolidated–undrained (UU) triaxial compression tests.

$$\sigma_{s_u} = \sqrt{\frac{1}{19}\sum_{1}^{20}(s_u - s_{u,\text{av}})^2} = 0.033 \text{ tsf} \qquad (13.4)$$

where s_u is the undrained shear strength and $s_{u,\text{av}}$ is the average undrained shear strength = 0.22 tsf. The *coefficient of variation* is the standard deviation divided by the expected value of a variable, which for practical purposes can be taken as the average:

$$\text{COV} = \frac{\sigma}{\text{average value}} \qquad (13.5)$$

where COV is the coefficient of variation, usually expressed in percent. Thus the coefficient of variation of the measured strengths in Table 13.3 is

$$\text{COV}_{s_u} = \frac{0.033}{0.22} = 15\% \qquad (13.6)$$

where COV_{s_u} is the coefficient of variation of the undrained strength data in Table 13.3.

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The coefficient of variation is a very convenient measure of scatter in data, or uncertainty in the value of the variable, because it is dimensionless. If all of the strength values in Table 13.3 were twice as large as those shown, the standard deviation of the values would be twice as large, but the coefficient of variation would be the same. The tests summarized in Table 13.3 were performed on high-quality test specimens using carefully controlled procedures, and the Bay mud at the Hamilton site is very uniform. The value of $COV_{s_u} = 15\%$ for these data is about as small as could ever be expected. Harr (1987) suggests that a representative value of $COV_{s_u} = 40\%$.

Estimates Based on Published Values

Frequently in geotechnical engineering, the values of soil properties are estimated based on correlations or on meager data plus judgment, and it is not possible to calculate values of standard deviation or coefficient of variation as shown above. Because standard deviations or coefficients of variation are needed for reliability analyses, it is essential that their values can be estimated using experience and judgment. Values of COV for various soil properties and in situ tests are shown in Table 13.4. These values may be of some use in estimating COVs for reliability analysis, but the values cover wide ranges, and it is not possible to use this type of information to make refined estimates of COV for specific cases.

The 3σ Rule

This rule of thumb, described by Dai and Wang (1992), uses the fact that 99.73% of all values of a normally distributed parameter fall within three standard deviations of the average. Therefore, if HCV is the highest conceivable value of the parameter and LCV is the lowest conceivable value of the parameter, these are approximately three standard deviations above and below the average value.

The 3σ rule can be used to estimate a value of standard deviation by first estimating the highest and lowest conceivable values of the parameter, and then dividing the difference between them by 6:

$$\sigma = \frac{\text{HCV} - \text{LCV}}{6} \tag{13.7}$$

where HCV is the highest conceivable value of the parameter and LCV is the lowest conceivable value of the parameter.

Consider, for example, how the 3σ rule can be used to estimate a coefficient of variation for a friction angle for sand that is estimated based on a correlation with standard penetration test blow count: For a value of $N_{60} = 20$, the *most likely value* (MLV) of ϕ' might be estimated to be 35°. However, no correlation is precise, and the value of ϕ' for a particular sand with an SPT blow count of 20 might be higher or lower than 35°. Suppose that the HCV was estimated to be 45°, and the LCV was estimated to be 25°. Then, using Eq. (13.7), the COV would be estimated to be

$$\sigma'_\phi = \frac{45° - 25°}{6} = 3.3° \tag{13.8}$$

and the coefficient of variation = 3.3°/35° = 9%.

Studies have shown that there is a tendency to estimate a range of values between HCV and LCV that is too small. One such study, described by Folayan et al. (1970), involved asking a number of geotechnical engineers to estimate the possible range of values of $C_c/(1 + e)$ for San Francisco Bay mud, with which

Table 13.4 Coefficients of Variation for Geotechnical Properties and In Situ Tests

Property or in situ test	COV (%)	References
Unit weight (γ)	3–7	Harr (1987), Kulhawy (1992)
Buoyant unit weight (γb)	0–10	Lacasse and Nadim (1997), Duncan (2000)
Effective stress friction angle (ϕ')	2–13	Harr (1987), Kulhawy (1992), Duncan (2000)
Undrained shear strength (S_u)	13–40	Kulhawy (1992), Harr (1987), Lacasse and Nadim (1997)
Undrained strength ratio (s_u/σ'_v)	5–15	Lacasse and Nadim (1997), Duncan (2000)
Standard penetration test blow count (N)	15–45	Harr (1987), Kulhawy (1992)
Electric cone penetration test (q_c)	5–15	Kulhawy (1992)
Mechanical cone penetration test (q_c)	15–37	Harr (1987), Kulhawy (1992)
Dilatometer test tip resistance (q_{DMT})	5–15	Kulhawy (1992)
Vane shear test undrained strength (S_v)	10–20	Kulhawy (1992)

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they all had experience. The data collected in this exercise are summarized below:

$$\text{Average value of } \frac{C_c}{1+e} \text{ estimate by experienced engineers} = 0.29$$

$$\text{Average value of } \frac{C_c}{1+e} \text{ from 45 laboratory tests} = 0.34$$

$$\text{Average COV of } \frac{C_c}{1+e} \text{ estimate by experienced engineers} = 8\%$$

$$\text{Average COV of } \frac{C_c}{1+e} \text{ from 45 laboratory tests} = 18\%$$

The experienced engineers were able to estimate the value of $C_c/(1 + e)$ for Bay mud within about 15%, but they underestimated the COV of $C_c/(1 + e)$ by about 55%.

Christian and Baecher (2001) showed that people (experienced engineers included) tend to be overconfident about their ability to estimate values, and therefore estimate possible ranges of values that are narrower than the actual range. If the range between the highest conceivable value (HCV) and the lowest conceivable value (LCV) is too small, values of coefficient of variation estimated using the 3σ rule will also be too small, introducing an unconservative bias in reliability analysis.

Based on statistical analysis, Christian and Baecher (2001) showed that the expected range of values in a sample containing 20 values is 3.7 times the standard deviation, and the expected range of values in a sample of 30 is 4.1 times the standard deviation. This information can be used to improve the accuracy of estimated values of standard deviation by modifying the 3σ rule. If the experience of the person making the estimate encompasses sample sizes in the range of 20 to 30 values, a better estimate of standard deviation would be made by dividing the range between HCV and LCV by 4 rather than 6:

$$\sigma = \frac{\text{HCV} - \text{LCV}}{4} \tag{13.9}$$

If Eq. (13.9) is used to estimate the coefficient of variation of ϕ', the value is

$$\sigma = \frac{45^\circ - 25^\circ}{4} = 5^\circ \tag{13.10}$$

and the coefficient of variation is $5^\circ/35^\circ = 14\%$.

With the 3σ rule it is possible to estimate values of standard deviation using the same amounts and types of data that are used for conventional deterministic geotechnical analyses. The 3σ rule can be applied when only limited data are available and when no data are available. It can also be used to judge the reasonableness of values of the coefficient of the variation from published sources, considering that the lowest conceivable value would be two or three standard deviations below the mean, and the highest conceivable value would be two or three standard deviations above the mean. If these values seem unreasonable, some adjustment of values is called for.

The 3σ rule uses the simple normal distribution as a basis for estimating that a range of three standard deviations covers virtually the entire population. However, the same is true of other distributions (Harr, 1987), and the 3σ rule is not tied rigidly to any particular probability distribution.

Graphical 3σ Rule

The concept behind the 3σ rule of Dai and Wang (1992) can be extended to a graphical procedure that is applicable to many situations in geotechnical engineering, where the parameter of interest, such as undrained shear strength, varies with depth. An examples is shown in Figure 13.1.

The steps involved in applying the graphical 3σ rule are as follows:

1. Draw a straight line or a curve through the data that represent the most likely average variation of the parameter with depth.
2. Draw straight lines or curves that represent the highest and lowest conceivable bounds on the data. These should be wide enough to include all valid data and an allowance for the fact that the natural tendency is to estimate such bounds too narrowly, as discussed previously. Note that some points in Figure 13.1 are outside the estimated highest and lowest conceivable lines, indicating that these data points are believed to be erroneous.
3. Draw straight lines or curves that represent the average plus one standard deviation and the average minus one standard deviation. These are one-third of the distance (or one-half of the distance) from the average line to the highest and lowest conceivable bounds.

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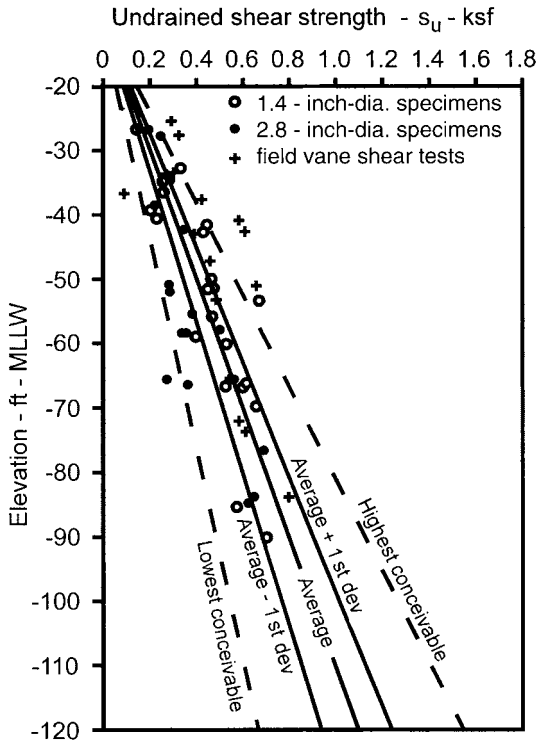


Figure 13.1 Example of graphical 3σ rule for undrained strength profile.

The average-plus-1σ and average-minus-1σ curves or lines are used in the Taylor series method described below in the same way as are parameters that can be represented by single values.

The graphical 3σ rule is also useful for characterizing strength envelopes for soils. In this case the quantity (shear strength) varies with normal stress rather than depth, but the procedure is the same. Strength envelopes are drawn that represent the average and the highest and lowest conceivable bounds on the data, as shown in Figure 13.2. Then average-plus-1σ and average-minus-1σ envelopes are drawn one-third of the distance (or one-half of the distance) from the average envelope to the highest and lowest conceivable bounds.

Using the graphical 3σ rule to establish average-plus-1σ and average-minus-1σ strength envelopes is preferable to using separate standard deviations for the strength parameters c and ϕ. Strength parameters (c and ϕ) are useful empirical coefficients that characterize the variation of shear strength with normal stress, but they are not of fundamental significance or interest by themselves. The important parameter is shear strength, and the graphical 3σ rule provides a straightforward means for characterizing the uncertainty in shear strength.

COEFFICIENT OF VARIATION OF FACTOR OF SAFETY

Reliability and probability of failure can be determined easily once the factor of safety and the coefficient of variation of the factor of safety (COV_F) have been determined. The value of factor of safety is determined in the usual way, using a computer program, slope stability charts, or spreadsheet calculations. The value of COV_F can be evaluated using the Taylor series method, which involves these steps:

1. Estimate the standard deviations of the quantities involved in analyzing the stability of the slope: for example, the shear strengths of the soils, the unit weights of the soils, the piezometric levels, the water level outside the slope, and the loads on the slope.
2. Use the Taylor series numerical method (Wolff, 1994; U.S. Army Corps of Engineers, 1998) to estimate the standard deviation and the coefficient of variation of the factor of safety, using these formulas:

$$\sigma_F = \sqrt{\left(\frac{\Delta F_1}{2}\right)^2 + \left(\frac{\Delta F_2}{2}\right)^2 + \cdots + \left(\frac{\Delta F_N}{2}\right)^2} \tag{13.11}$$

$$\text{COV}_F = \frac{\sigma_F}{F_{\text{MLV}}} \tag{13.12}$$

where $\Delta F_1 = (F_1^+ - F_1^-)$. F_1^+ is the factor of safety calculated with the value of the first parameter increased by one standard deviation from its most likely value, and F_1^- is the factor of safety calculated with the value of the first parameter decreased by one standard deviation.

In calculating F_1^+ and F_1^-, the values of all of the other variables are kept at their most likely values. The other values of ΔF_2, ΔF_3, . . . Δ, F_N are calculated by varying the values of the other variables by plus and minus one standard deviation from their most likely values. F_{MLV} in Eq. (13.12) is the most likely value of factor of safety, computed using most likely values for all the parameters.

Substituting the values of ΔF into Eq. (13.11), the value of the standard deviation of the factor of safety (σ_F) is computed, and the coefficient of variation of the factor of safety (COV_F) is computed using Eq. (13.12). With both F_{MLV} and COV_F known, the probability of failure (P_f) can be determined using Table

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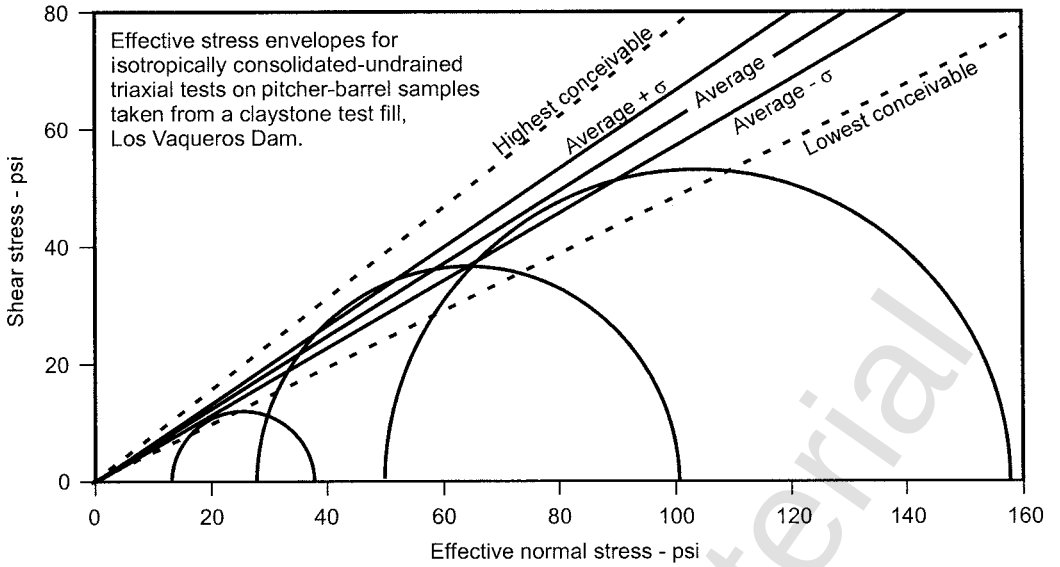


Figure 13.2 Example of graphical 3σ rule for shear strength envelope.

13.5, Figure 13.3, or the reliability index, as explained below.

Table 13.5 and Figure 13.3 assume that the factor of safety is lognormally distributed, which seems reasonable because calculating the factor of safety involves many multiplication and division operations. The central limit theorem indicates that the result of adding and subtracting many random variables approaches a normal distribution as the number of operations increases. Since multiplying and dividing amounts to adding and subtracting logarithms, it follows that the factor of safety distribution can be approximated by a lognormal distribution. Thus, although there is no proof that factors of safety are lognormally distributed, it is at least a reasonable approximation. The assumption of a lognormal distribution for factor of safety does not imply that the values of the individual variables must be distributed lognormally. It is not necessary to make any particular assumption concerning the distributions of the variables to use this method.

RELIABILITY INDEX

The *reliability index* (β) is an alternative measure of safety, or reliability, which is uniquely related to the probability of failure. The value of β indicates the number of standard deviations between $F = 1.0$ (failure) and F_{MLV}, as shown in Figure 13.4. The usefulness of β lies in the fact that probability of failure and reliability are uniquely related to β, as shown in Figure 13.5. The *lognormal reliability index*, β_{LN}, can be determined from the values of F_{MLV} and COV_F using Eq. (13.13):

$$\beta_{LN} = \frac{\ln (F_{MLV}/\sqrt{1 + COV_F^2})}{\sqrt{\ln(1 + COV_F^2)}} \qquad (13.13)$$

where β_{LN} is the lognormal reliability index, F_{MLV} the most likely value of factor of safety, and COV_F the coefficient of variation of factor of safety.

The relationship between β and P_f shown in Figure 13.5 is called the *standard cumulative normal distribution function,* which can be found in many textbooks on probability and reliability. Values of P_f corresponding to a given value of β can be calculated using the NORMSDIST function in Excel. The argument of this function is the reliability index, β_{LN}. In Excel, under "Insert Function," "Statistical," choose "NORMSDIST" and type the value of β_{LN}. The result is the reliability, R. For example, for $\beta_{LN} = 2.32$, the result is 0.9898, which corresponds to $P_f = 0.0102$. Table 13.5 and Figures 13.3 and 13.5 were developed using this Excel function.

PROBABILITY OF FAILURE

Once the most likely value of factor of safety (F_{MLV}) and the coefficient of variation of factor of safety (COV_F) have been evaluated, the probability of failure (P_f) can be determined in any of the following ways:

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Table 13.5 Probabilities of Failure (%) Based on Lognormal Distribution of *F*

	COV_F = coefficient of variation of factor of safety								
F_{MLV}[a]	10%	12%	14%	16%	20%	25%	30%	40%	50%
1.05	33.02	36.38	38.95	41.01	44.14	47.01	49.23	52.63	55.29
1.10	18.26	23.05	26.95	30.15	35.11	39.59	42.94	47.82	51.37
1.15	8.83	13.37	17.53	21.20	27.20	32.83	37.10	43.24	47.62
1.20	3.77	7.15	10.77	14.29	20.57	26.85	31.76	38.95	44.05
1.25	1.44	3.54	6.28	9.27	15.20	21.68	26.98	34.95	40.66
1.30	0.49	1.64	3.49	5.81	11.01	17.30	22.75	31.26	37.48
1.35	0.15	0.71	1.86	3.53	7.83	13.66	19.06	27.88	34.49
1.40	0.04	0.29	0.95	2.08	5.48	10.69	15.88	24.80	31.70
1.50	0.00	0.04	0.23	0.67	2.57	6.38	10.85	19.49	26.69
1.60	0.00	0.01	0.05	0.20	1.15	3.71	7.29	15.21	22.40
1.70	0.00	0.00	0.01	0.06	0.49	2.11	4.84	11.81	18.75
1.80	0.00	0.00	0.00	0.01	0.21	1.18	3.18	9.13	15.67
1.90	0.00	0.00	0.00	0.00	0.08	0.65	2.07	7.03	13.08
2.00	0.00	0.00	0.00	0.00	0.03	0.36	1.34	5.41	10.91
2.20	0.00	0.00	0.00	0.00	0.01	0.10	0.56	3.19	7.59
2.40	0.00	0.00	0.00	0.00	0.00	0.03	0.23	1.88	5.29
2.60	0.00	0.00	0.00	0.00	0.00	0.01	0.09	1.11	3.70
2.80	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.66	2.60
3.00	0.00	0.00	0.00	0.00	0.00	0.00	0.02	0.39	1.83

[a] F_{MLV}, factor of safety computed using most likely values of parameters.

1. Using Table 13.5
2. Using Figure 13.3
3. Using Figure 13.5, with β_{LN} computed using Eq. (13.13)
4. Using the Excel function NORMSDIST, with β_{LN} computed using Eq. (13.13)

Interpretation of Probability of Failure

The event whose probability is described as the *probability of failure* is not necessarily a catastrophic failure. In the case of shallow sloughing of a slope, for example, failure very likely would not be catastrophic. If the slope could be repaired easily and there were no serious secondary consequences, shallow sloughing would not be catastrophic. However, a slope failure that would be very expensive to repair, or that would have the potential for delaying an important project, or that would involve threat to life, would be catastrophic. Although the term *probability of failure* would be used in both of these cases, it is important to recognize the different nature of the consequences.

In recognition of this important distinction between catastrophic failure and less significant performance problems, the Corps of Engineers uses the term *probability of unsatisfactory performance* (U.S. Army Corps of Engineers, 1998). Whatever terminology is used, it is important to keep in mind the real consequences of the event analyzed and not to be blinded by the word *failure* where the term *probability of failure* is used.

Probability of Failure Criteria

There is no universally appropriate value of probability of failure. Experience indicates that slopes designed in accord with conventional practice often have a probability of failure in the neighborhood of 1%, but like factor of safety, the appropriate value of P_f should depend on the consequences of failure.

One important advantage of probability of failure is the possibility of judging an acceptable level of risk based on the potential cost of failure. Suppose, for example, that two alternative designs for the slopes on a project are analyzed, with these results:

- *Case A.* Steep slopes, construction and land costs = \$100,000, $P_f = 0.1$.
- *Case B.* Flat slopes, construction and land costs = \$400,000, $P_f = 0.01$.

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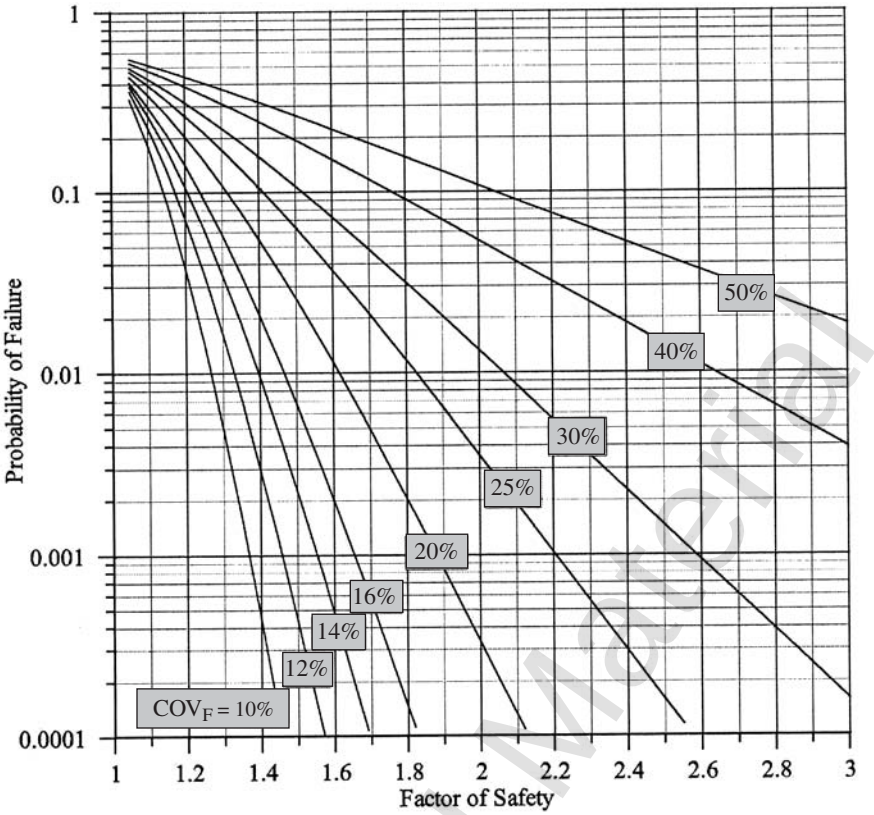


Figure 13.3 Probabilities of failure based on lognormal distribution of F.

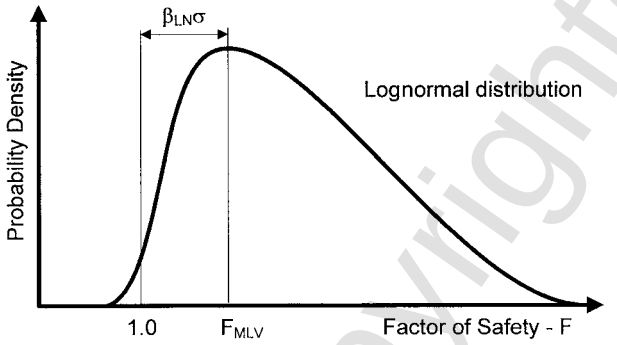


Figure 13.4 Relationship of β_{LN} to probability distribution.

Suppose further that the consequences of failure are estimated to be the same in either case, \$5,000,000, considering primary and secondary consequences of failure. In case A, the total cost of construction, land, and probable cost of failure is \$100,000 + (0.1)(\$5,000,000) = \$600,000. In case B, the total cost is \$400,000 + (0.01)(\$5,000,000) = \$450,000. Considering the probable cost of failure, as well as construction and land costs, case B is less costly overall.

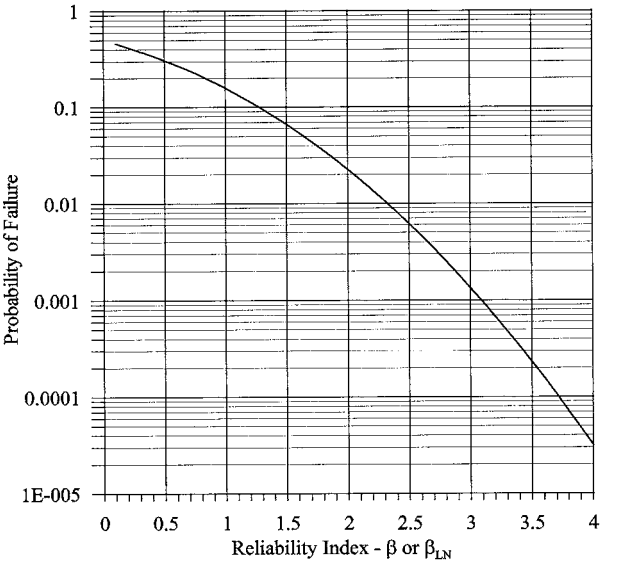


Figure 13.5 Variation of P_f with β.

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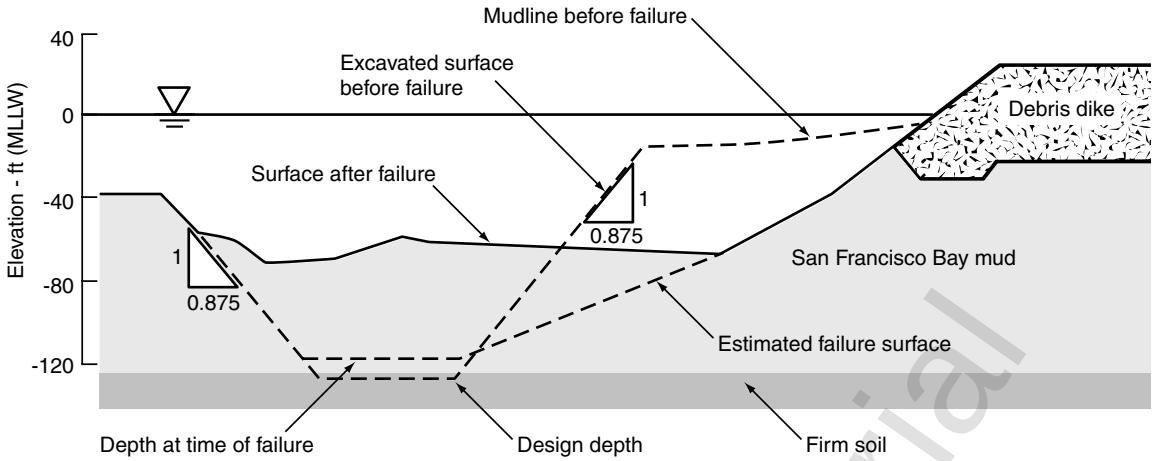


Figure 13.6 Underwater slope failure in San Francisco Bay.

Even without cost analysis, P_f may provide a better basis for judging what is an acceptable risk than does factor of safety. Many people may find that comparing one chance in 10 with one chance in 100 provides a more understandable basis for decision than does comparing a factor of safety of 1.3 with a factor of safety of 1.5.

Example. In August 1970, a trench about 100 ft deep was excavated underwater in San Francisco Bay. The trench was to be filled with sand to stabilize the adjacent area and reduce seismic deformations at a new lighter aboard ship (LASH) terminal. The trench slopes were made steeper than was the normal practice in order to reduce the volume of excavation and fill. As shown in Figure 13.6, the slopes were excavated at an inclination of 0.875 horizontal to 1.0 vertical.

On August 20, after a section of the trench about 500 ft long had been excavated, the dredge operator found that the clamshell bucket could not be lowered to the depth from which mud had been excavated only hours before. Using the side-scanning sonar with which the dredge was equipped, four cross sections made within two hours showed that a failure had occurred that involved a 250-ft-long section of the trench. The cross section is shown in Figure 13.6. Later, a second failure occurred, involving an additional 200 ft of length along the trench. The rest of the 2000-ft-long trench remained stable for about four months, at which time the trench was backfilled with sand. Additional details regarding the failure can be found in Duncan and Buchignani (1973).

Figure 13.1 shows the variation of undrained strength of the Bay mud at the site, and the average, average + σ and average − σ lines established by Duncan (2000) for a reliability analysis of the slope. The average buoyant unit weight of the Bay mud was 38 pcf and the standard deviation was 3.3 pcf, based on measurements made on undisturbed samples.

Factors of safety calculated using average values of strength and unit weight (F_{MLV}) and using average $+\sigma$ and average $-\sigma$ values are shown in Table 13.6. The ΔF value for variation in Bay mud strength is 0.31, and the ΔF value for unit weight variation is 0.20. The ΔF due to strength variation is always significant, but

Table 13.6 Reliability Analysis for 0.875 Horizontal on 1.0 Vertical Underwater Slope in San Francisco Bay Mud

Variable	Values	F	ΔF
Undrained shear strength Buoyant unit weight	Average line in Figure 13.1 $\gamma_{b(\text{av})}$ = 38 pcf	F_{MLV} = 1.17	—
Undrained shear strength	Average + σ line in Figure 13.1	F^+ = 1.33	0.31
	Average − σ line in Figure 13.1	F^- = 1.02	
Buoyant unit weight	Average + σ = 41.3 pcf	F^+ = 1.08	0.20
	Average − σ = 34.7 pcf	F^- = 1.28	

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Table 13.7 Summary of Analyses of LASH Terminal Trench Slope

Case	Slope (H on V)	F_{MLV}	COV_F[a] (%)	P_f (%)	Trench volume[b] (yd^3)
As constructed	0.875 on 1.0	1.17	16%	18%	860,000
Less-steep A	1.25 on 1.0	1.3	16%	6%	1,000,000
Less-steep B	1.6 on 1.0	1.5	16%	1%	1,130,000

[a]The value of COV_F is the same for all cases because COVs of strength and unit weight are the same for all cases.
[b]For the as-constructed case, an additional 100,000 yd^3 of slumped material had to be excavated after the failure.

it is unusual for the ΔF due to unit weight variation to be as large as it is in this case. Its magnitude in this case is due to the fact that the buoyant unit weight is so low, only 38 pcf. Therefore, the variation by ± 3.3 pcf has a significant effect.

The standard deviation and coefficient of variation of the factor of safety are calculated using Eqs. (13.11) and (13.12):

$$\sigma_F = \sqrt{\left(\frac{0.31}{2}\right)^2 + \left(\frac{0.20}{2}\right)^2} = 0.18 \quad (13.14)$$

$$COV_F = \frac{0.18}{1.17} = 16\% \quad (13.15)$$

The probability of failure corresponding to these values of F_{MLV} and COV_F can be determined using any of the four methods discussed previously. A value of $P_f = 18\%$ was determined using the Excel function NORMSDIST. Such a large probability of failure is not in keeping with conventional practice. Although it appeared in 1970, when the slope was designed, that the conditions were known well enough to justify using a very low factor of safety of 1.17, the failure showed otherwise. Based on this experience, it is readily apparent that such a low factor of safety and such a high probability of failure exceed the bounds of normal practice. The probability of failure was computed after the failure (Duncan, 2000) and was not available to guide the design in 1970. In retrospect, it seems likely that knowing that the computed probability of failure was 18% might have changed the decision to make the trench slopes so steep.

The cost of excavating the mud that slid into the trench, plus the cost of extra sand backfill, was approximately the same as the savings resulting from the use of steeper slopes. Given the fact that the expected savings were not realized, that the failure caused great alarm among all concerned, and that the confidence of the owner was diminished as a result of the failure, it is now clear that using 0.875 (horizontal) on 1 (vertical) slopes was not a good idea.

Further analyses have been made to determine what the probability of failure would have been if the inboard slope had been excavated less steep. Two additional cases have been analyzed, as summarized in Table 13.7. The analyses were performed using the chart developed by Hunter and Schuster (1968) for shear strength increasing linearly with depth, which is given in the Appendix. The factors of safety for the less-steep alternatives A and B are in keeping with the factor of safety criteria of the Corps of Engineers, summarized in Table 13.2.

A parametric study such as the one summarized in Table 13.7 provides a basis for decision making and for enhanced communication among the members of the design team and with the client. The study could be extended through estimates of the costs of construction for the three cases and estimates of the potential cost of failure. This would provide a basis for the design team and clients to decide how much risk should be accepted. This type of evaluation was not made in 1970 because only factors of safety were computed to guide the design.

Recapitulation

- Uncertainty about shear strength is usually the largest uncertainty involved in slope stability analyses.
- The most widely used and most generally useful definition of factor of safety for slope stability is

$$F = \frac{\text{shear strength of the soil}}{\text{shear stress required for equilibrium}}$$

- The value of the factor of safety used in any given case should be commensurate with the uncertainties involved in its calculation and the consequences of failure.
- Reliability calculations provide a means of evaluating the combined effects of uncertainties and a means of distinguishing between conditions where uncertainties are particularly high or low.

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- Standard deviation is a quantitative measure of the scatter of a variable. The greater the scatter, the larger the standard deviation. The coefficient of variation is the standard deviation divided by the expected value of the variable.
- The 3σ rule can be used to estimate a value of standard deviation by first estimating the highest and lowest conceivable values of the parameter and then dividing the difference between them by 6. If the experience of the person making the estimate encompasses sample sizes in the range of 20 to 30 values, a better estimate of standard deviation can be made by dividing by 4 rather than 6.

- Reliability and probability of failure can readily be determined once the factor of safety (F_{MLV}) and the coefficient of variation of the factor of safety (COV_F) have been determined, using the Taylor series numerical method.
- The event whose probability is described as the probability of failure is not necessarily a catastrophic failure. It is important to recognize the nature of the consequences of the event and not to be blinded by the word *failure*.
- The principal advantage of probability of failure, in contrast with factor of safety, is the possibility of judging acceptable level of risk based on the estimated cost and consequences of failure.

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